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Modeling individual tree mortality for crimean pine plantations

Mehmet Misir*, Nuray Misir and Hakki Yavuz

Faculty of Forestry, Karadeniz Technical University, Trabzon-610 80, Turkey

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Abstract: *Individual tree mortality model was developed for crimean pine (Pinus nigra subsp. pallasiana) plantations in Turkey. Data came from 5 year remeasurements of the permanent sample plots. The data comprises of 115 sample plots with 5029 individual trees. Parameters of the logistic equation were estimated using weighted nonlinear regression analysis. Approximately 80% of the observations were used for model development and 20% for validation. The explicatory variables in the model were ratio of diameter of the subject tree and basal area mean diameter of the sample plot as measure of competition index for individual trees, basal area and site index. All parameter estimates were found highly significant (p<0.001) in predicting mortality model. Chi-square statistics indicate that the most important variable is* $d / \bar{d}_q$ *, the second most important is site index, and the third most important predictor is stand basal area. Examination of graphs of observed vs. predicted mortality rates reveals that the mortality model is well behaved and match the observed mortality rates quite well. Although the phenomenon of mortality is a stochastic, rare and irregular event, the model fit was fairly good. The logistic mortality model passed a validation test on independent data not used in parameter estimation. The key ingredient for obtaining a good mortality model is a data set that is both large and representative of the population under study and the data satisfy both requirements. The mortality model presented in this paper is considered to have an appropriate level of reliability.*

Key words: *Growth model, Individual trees, Mortality, Logistic function*

Introduction

Crimean pine *(Pinus nigra* sub sp. *pallasiana)* is one of the most important plant species in Turkey. Large scale plantations establishment of crimean pine in Turkey were started in 1955. These plantations are estimated to cover about 460 hectares of land. Thus, it is important that development of growth and yield models for these areas provide a tool for forest management.

To account for mortality, two considerations, the self thinning limit and the probability of a tree to die during the coming growth period, have been found. The model for self thinning limit is based on the relationship between the total number of trees and the mean tree size of a fully stocked stand. On a log scale, the relationship has been found to be linear and the slope of the line is near -3/2, but varies according to species shade tolerance, locality and the site type (Zeide, 1993; Rautiainen, 1999). In this work, mortality model estimated the probability of a tree to die during the coming growth period was dealt.

Mortality remains one of the least understood component of growth and yield estimation (Hamilton, 1986). The key to a tree's survival or mortality is its genetic make up and its environment (Spurr and Barnes, 1980). Growth models almost universally ignore a tree's genetic status (Monserud and Rehfeldt, 1990), as well as important environmental factors such as climatic extremes (*e.g.* wind, drought, killing frost), insects and diseases. Great detail is paid to environmental competition arising from neighboring trees, however (Buchman *et al.*, 1983), as well as measurable gross physical features of the tree and site. Perhaps mortality would appear less stochastic if relevant environmental variables were measured on permanent plots and if the genetic status of the trees could be characterized.

The literature on modeling mortality of forest trees is not small, but successes are rare. Realistically, mortality models hope to capture the average rate of mortality and relate it to a few reliable and measurable size or site characteristics. The key then is a large and representative sample of remeasured trees so that a rare eventmortality can be observed frequently enough, to predict it accurately. A representative sample must reflect both the full range in site variability as well as the diversity of management treatments in a given population (Hamilton, 1980). Because mortality data are most reliable and efficiently obtained from permanent plots, researchers are often forced to live with the limitations of the underlying permanent plot network. As a result, many studies rely on data from either unthinned plots (Zhang *et al.*, 1997) or only lightly thinned plots (Dursky, 1997). Even if the data certain treatments, the permanent plots often are clustered spatially and do not necessarily represent the average and dispersion of stands in the entire region of interest.

The most common methodology for modeling individual tree mortality is statistical. Generally, the parameters of a flexible non linear function bounded by 0 and 1 are estimated using weighted non linear regression or a multivariate maximum likelihood procedure (Neter and Maynes, 1970). Although most cumulative distribution functions will work, the most popular is the logistic or logit (Hamilton and Edwards, 1976; Buchman, 1979;

****Corresponding author:*** E-Mail: mmisir@ktu.edu.tr, Tel.: + 90 462 377 28 75, Fax: + 90 462 325 74 99

Hamilton, 1986; Vanclay, 1995; Monserud and Sterba, 1999). Other applications have used the Weibull (Somers *et al.*, 1980), the Gamma (Kobe and Coates, 1997), the Richard's function (Buford and Hafley, 1985), the Exponential (Moser, 1972) and the Normal or Probit (Finney, 1971). Monserud (1976), found that both the probit and the logit produced similar results, even though the underlying functional forms are quite different.

Until now neither an individual tree model nor any mortality model had been developed in Turkey. The aim of the present study was to develop model for individual tree mortality based on permanent sample plots. For this aim in this work, mortality model estimated the probability of a tree to die aduring the coming growth period. The model should be applicable to even aged forest as well as forest with mixed and pure species composition. Since the mortality models also should be applicable to large scale forestry scenario analysis in practical management planning, the modeling was restricted to include only exploratory variables that directly or indirectly are available from practical inventories.

Materials and Methods

Data from remeasurement in 2001 were collected from 115 sample plots ranging in ages from 3 to 58 years. Sample plantations were taken from seven Forest Conservancies, Regional Forest Headquarters in Turkey. Plots were thinned and located on three different sites. Data for this study were obtained from crimean pine plantations which have the same spacing (3x1.5 meters). Rectangular plots area varied from 165 m^2 to 600 m^2.

Plantations were stratified into 10 year age classes and sampled with an effort to equal allocation of at least three sample plots to each age group. For each age group, effort was also made to include the full range of site conditions (from poorest to best). The sites ranged in site index (at an index age of 40 years) from 5.80 m. to 22.41 m. The site index for each site was determined using the age/dominant height model (Misir, 2004). The mean site index for sample plots was 12.52 m.

For each plots, all trees were measured for diameter at breast height, diameter at stump level, total height, age and age at breast height. At for each plots slope, altitude aspect was also measured.

The plots were remeasured in 2001 at intervals ranging from about 5 to 10 years. This resulted in a total of 9522 diameter/height observations from 5029 trees. At each measurement time, stand characteristics were computed from individual tree measurements in the stands. The values for volume per hectare (*V*), basal area (*BA*), basal area mean diameter ($\bar{d}_q$), mean height weighted by basal area ($\bar{h}_q$), Relative Stand Density ($RSD = BA/\sqrt{\bar{d}_q}$ where BA and $d_{\bar{g}}$ stand basal area (m^2/ha) and mean diameter (cm) (Curtis *et al.*, 1981), and number of trees per hectare (*N*) in Table 1 were based on individual trees measured on the sample plots. In addition, non spatial a competition index (*BAL*, the summarized basal area for all greater than the subject) computed for each individual tree and some individual tree characteristics were given in Table 1.

Table - 1: Mean, S.D. range of main characteristics in the study material

	Minimumu	Maximum	Mean	S.D.
d (cm)	2.2	45.0	12.98	7.288
H (m)	1.24	21.90	8.70	3.537
$\bar{d}_q$ (cm)	0.51	32.64	12.87	6.870
$\bar{h}_q$ (m)	0.42	18.53	6.62	4.291
Hdom (m)	0.51	22.33	7.47	4.579
A (year)	3	58	27	10.310
BA (m^2/ha)	1.71	50.56	19.03	11.647
RSD	0.28	10.29	3.73	2.388
V (m^3/ha)	0.00	704.05	371.05	131.929
Number of trees	407	1958	1125	419
BAL	0.0	50.56	15.11	12.543
S (m)	5.80	22.41	12.43	2.824

d : Diameter at breast height (cm)
S : Site index, dominant height at breast height age 40 years (m)
A : Age (years)
H_{dom} : Dominant height, mean height of the 100 thickest trees ha^{-1}(m)
V : Volume (m^3 ha^{-1})
BA : Basal area (m^2 ha^{-1})
$\bar{d}_q$: Basal area mean diameter (cm)
$\bar{h}_q$: Mean height weighted by basal area (m)
N : Number of trees (ha^{-1})
RSD : Stand density
S.D. : Standard deviation

Each tree was classified as alive or as dead at the time of plot establishment (as well as remeasured). Dead trees were defined as (i) standing or (ii) fallen trees with no green branches. At remeasurement (measurement in 2001), all trees were alive on 20 (17.1%) sample plots (Table 2). Out of 5029 living trees at the time of plot establishment, 851 trees were dead when they were remeasured (Table 3).

Table - 2: Number of dead trees by number of sample plots

No. of dead trees	0	1	2	3	4	5	6	7	8	9	>10
No. of sample plots	20	13	9	7	10	10	7	6	5	2	26

Table - 3: Total number of trees and number and percent of dead trees

Total number of trees	Dead trees	
	Nunber	%
5029	851	16.9

Individual tree mortality is a discrete event, *i.e.* only the values 0 (live) or 1 (dead) may occur. Although a number of cumulative distribution functions may work as the classification function when natural mortality is modeled (*e.g.* Buford and Hafley, 1985), the logistic function is probably the most widely used in models for individual trees (Monserud, 1976; Buchman *et al.*, 1983; Hamilton, 1986; Vanclay, 1991; Avila and Burkhart, 1992; Tuhus, 1997; Monserud and Sterba, 1999).

The candidate variables for the mortality models were numerous and diverse. Hamilton (1986), classified such variables in four groups which were measures of individual tree size, tree competition, tree vitality and measures of stand density. Several mortality models include diameter growth rate as a measure for vitality (Monserud, 1976; Buchman *et al.*, 1983; Hamilton, 1986) or as a substitute crown size (Avila and Burkhart, 1992; Zhang *et al.*, 1997; Monserud and Sterba, 1999).

The candidate variables of the present study were divided into tree groups: (i) measures of individual tree size (ii) measures of individual tree competition and (iii) measures at the stand level of structure, density and productivity.

The only variable available for describing individual tree size was diameter at breast height (*d*). Usually a linear function of diameter is not sufficient to describe mortality. Several possibilities for transformation of this variable exist and have been applied (Hamilton, 1986; Ojansuu *et al.*, 1991; Vanclay, 1991; Tuhus, 1997; Monserud and Sterba, 1999). In addition to a linear relationship d^{-1}, d^2 were tested in the present work.

Examples of competition indices used in mortality models are $d/\bar{d}_q$ *i.e.* ratio of diameter of the subject tree and basal area mean diameter of the sample plot (Hamilton, 1986; Avila and Burkhart, 1992; Burgman *et al.*, 1994), *BAL*, *i.e.* the summarized basal area for all greater than the subject tree (Ojansuu *et al.*, 1991; Monserud and Sterba, 1999), *RS*, *i.e.* relative status of the tree expressed as the relative position on the cumulative basal area distribution (the biggest tree has *RS*=0, while the smallest has *RS*=1) (Vanclay, 1991) and h/H_{dom}, *i.e.* ratio of height of subject tree and dominant height of the sample plot (Avila and Burhart, 1992; Zhang *et al.*, 1997). In the present study, $d/\bar{d}_q$ and *BAL* were tested. In addition to proportion basal area (*PBA*) were tested as an exampled stand structure.

Several possibilities exist to describe stand density. Hamilton (1986); Ojansuu *et al.* (1991); Vanclay (1991); Tuhus (1997); Mabvurira and Miina (2002); Eid and Tuhus (2001), all of whom used *BA* and Burgman *et al.* (1994) who used *N*, have provided examples of models with stand density parameters as explicatory variables. Since *N* and *BA* were directly determined and did not rely on functional relationships as opposed to volume (*V*), only these two variables were selected for testing in the present study.

Based on the discussion above, the following mortality model was hypothesized :

$$P(die) = \frac{\exp(\beta_0 + \beta_1 \times t_1 + \beta_2 \times t_2 + \beta_3 \times S_1 + \beta_4 \times S_2 + \beta_5 \times S_3)}{1 + \exp(\beta_0 + \beta_1 \times t_1 + \beta_2 \times t_2 + \beta_3 \times S_1 + \beta_4 \times S_2 + \beta_5 \times S_3)} \quad (1)$$

where t_1 is the individual tree size (d, d^{-1}, d^2), t_2 is the individual tree competition ($d/\bar{d}_q$, *BAL*), S_1 is the stand density (*N, BA*), S_2 is the stand structure (*PBA*) and S_3 is the stand productivity (*S*).

The validation of a model should involve independent data. For the present work data were partitioned in two groups, one for model development and one for validation. Many solutions for partitioning of such data are at hand, both with respect to method and with respect to number of observations in the respective data sets (Vanclay, 1994). In order to secure the range of site and stand conditions in both data sets, simple random sampling was used in the present study. The data set used for model development comprised approximately 80% of the observations (4023), while the remaining 20 % of the observations (1006) were used for validation. Although the number of observations determined for model development was made relatively large in order to provide sufficient data for the model development phase, the number of observations in the test data still should be large enough for validation and appropriate statistical test.

Nonlinear regression was used to estimate the parameters of the hypothesized model. This procedure produces weighted least squares estimates of the parameters of a non linear model. For each non-linear model to be analyzed, the names and starting values for the parameters to be estimated, the model and the partial derivatives of the model with respect to each parameter, were specified. The significance of the parameter estimates was tested by means of Z = β/ASE, where β is the parameter estimate and ASE is the asymptotic standard error (Agresti, 1996; Eid and Tuhus, 2001).

The validation was based on two different concepts. First, predicted and observed mortality were compared by visually studying deviations over the explicatory variables included in the models. The model development data set as well as the test data set was compared in this way. Secondly, the test data set was started according to certain attributes not included as explicatory variables in the mortality models, and then used for comparisons of predicted and observed mortality over different classes within the attributes. The deviations between predicted and observed values were tested by means of Pearson Chi-Squared Statistics (Agresti, 1996; Eid and Tuhus, 2001). Chi-Squared values for classes within an attribute were calculated as:

$$\chi^2 = \frac{(N_{obs} - N_{pred})^2}{N_{pred}} \qquad (2)$$

where N_{obs} is the number of observed dead trees in a class and N_{pred} is the number of predicted dead trees in a class. Large chi-squared values provided evidence of lack of fit.

Results and Discussion

All parameters were highly significant (p<0.001) in predicting tree mortality (Table 4). Ratio of diameter of the subject tree and basal area mean diameter of the sample plot ($d/\overline{d}_q$), basal area and site index were significant in predicting mortality for crimean pine plantations (p<0.001).

The mortality rate in crimean pine plantations is the best explained by stand basal area, site index and the relative size of a tree (Eq. 1). The parameter estimates for the mortality model are significant (p<0.001) (Table 5). The mortality in crimean pine plantations decreased when diameter increased. Accordingly, the larger the value of the exponent, the greater is the mortality. This means that the denser the plantation (stand) and the more suppressed the tree, the greater is the mortality rate. The mortality rate is lower on good sites than on poor sites.

Fig. 1, 2 and 3 show predicted and observed mortality plotted over *d, BA* and *S,* respectively. The predicted mean values of the figures were calculated using actual values of the explicatory variables for each observation. In general, the mortality fitted well over the explicatory variables in the model data set.

The average predicted and observed numbers of dead trees in the test data set were 4.2 and 3, respectively, *i.e.* a relative deviation of 40 % (Table 5).

The chi squared statistics gave no evidence of lack of fit between predicted and observed values. Since number of trees ha^{-1} (*N*) was hypothesized as an explicatory variable, but failed in predicting mortality rates, the test data were stratified according to *N* and investigated more thoroughly (Table 6). No evidence of lack of fit was detected over *N*, however.

Table - 4: Estimated parameters and standard errors for mortality model (Eq. 1)

Variable	Estimate	Standard error
Intercept	-3.3614861***	0.111349
d/	-3.406156***	0.128080
BA (m^2 ha^{-1})	0.001230***	0.0004109
S (m)	0.004647***	0.0008986

R^2 =0.42 ***: p<0.001

Table - 5: Predicted and observed mortality over tree species in test data set

Total number of trees	Predicted No	Predicted %	Observed No	Observed %	Deviation %	χ^2
1006	4.2	0.42	3	0.30	40	0.34ns[b]

[b] Significant level, ns: not significant

Table - 6: Predicted and observed mortality over classes of number of trees ha^{-1} [a]

N (ha^{-1})	Total no. of trees	Number of predicted	Dead trees observed	Deviation %	χ^2	χ^2(Accumulated)
0-500	35	0.1	0	0	0.10000	0.10000
501-1000	337	1.4	1	40	0.11429	0.21429
1001-1500	394	1.8	2	-10	0.02222	0.23651
1501-2000	240	0.9	0	0	0.90000	1.13651ns[b]
Σ	1006	4.2	3			

[a] Test data set, [b] Significant level, ns: not significant

Mortality of individual trees is a stochastic, rare, and irregular phenomenon. Many mortality models for individual trees include three or fewer explicatory variables (Monserud, 1976;

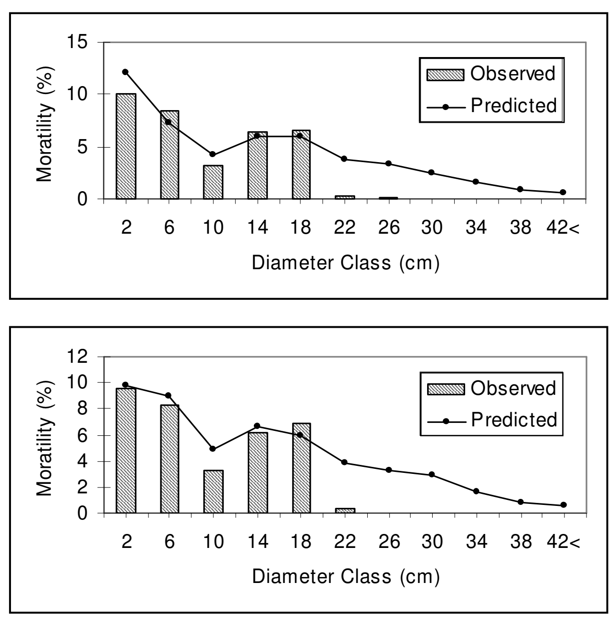

Fig. 1: Predicted and observed mortality over diameter classes for model data set (upside) and test data set (bottom side)

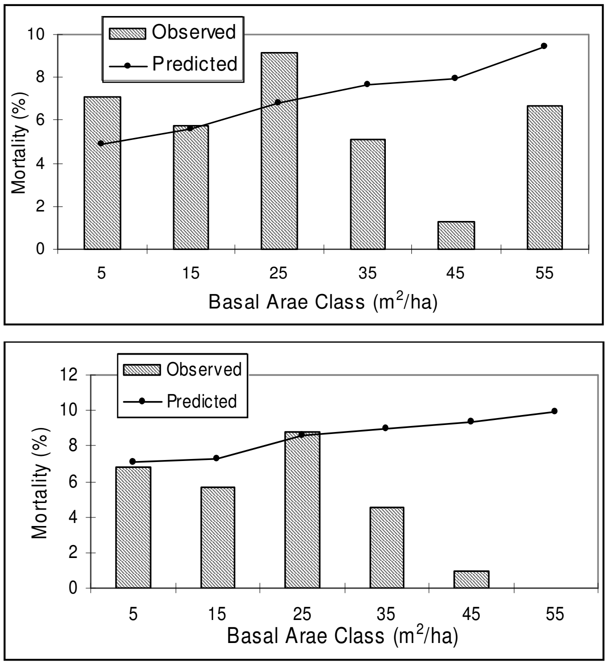

Fig. 2: Predicted and observed mortality over *BA* classes for model data (upside) set and test data set (bottom side)

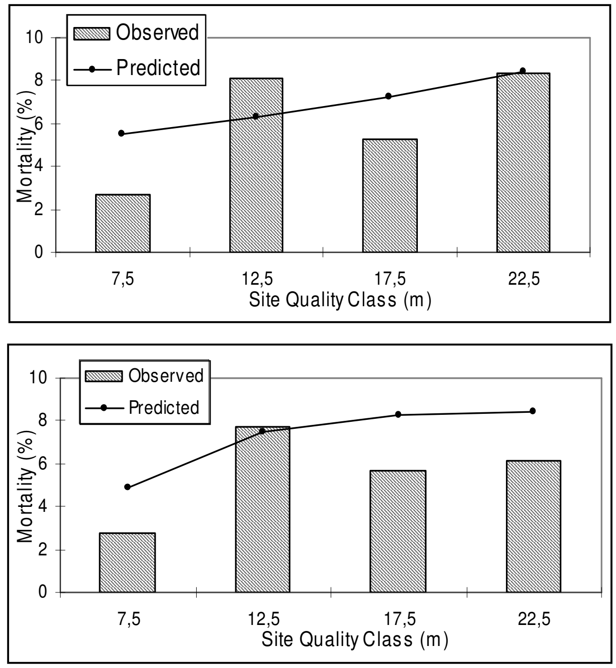

Fig. 3: Predicted and observed mortality over site index classes for model data set (upside) and test data set (bottom side)

Buchman *et al.*, 1983; Ojansuu *et al.*, 1991; Avila and Burkhart, 1992; Monserud and Sterba, 1999; Eid and Tuhus, 2001). Thus, it was as expected when several candidate variables of the hypothesized mortality model were excluded when it came to the final models (Table 4).

The hyperbolic d^{-1} founded to be significant in many mortality models was failed in the present study. In addition to a model with a linear relationship between *d* and mortality was tested but parameter estimate was insignificant. However, the model gave decreased mortality with increased diameter. Monserud and Sterba (1999), hypothesized on increased mortality rate for very large diameters due to senescence of overmature trees, included d^2 in the model and found it significant in predicting mortality for Norway spruce. No signs of such effects were found in the present study.

The competition indices *BAL* and *d/*were tested in predicting mortality but only ratio of diameter of the subject tree and basal area mean diameter of the sample plot means that the relative size of a tree was found significant ($p<0.001$). *BAL* gives on insignificant parameter estimate.

N and *BA* were tested as measures for stand density but only *BA* was found significant. *N* failed as explicatory variable because the parameter estimate was insignificant. Several previous models include a competition index and a measure for density at the same time (Hamilton, 1986; Vanclay, 1991; Ojansuu *et al.*, 1991; Burgman *et al.*, 1994). In the present data the density seemed too much correlated with mortality. When the predicted and observed mortality of the test data set were compared over different classes for *N*, no evidence of lack of fit was found (Table 6).

Site index (S) was found significant in predicting mortality *i.e.* the better the site index the greater the mortality rate (Table 4). The model data indicated a more rapidly increasing mortality for the best side indices.

The average deviation between predicted and observed mortality in the test data was 40% (Table 5). No evidence of lack of fit was found. The same holds true when predicted and observed mortality were compared over different densities (Table 6).

Plots that had been subjected to any harvesting operation between the measurements were included from the data material because of sufficient information about treatments. If the harvest on these plots was a result of "regular" management practices, there were no problems related to the inclusion. However, if the harvest was a result of an extraordinary situation (*e.g.* disease, wind damage), exclusion of the plots may have lead to an overestimated mortality rate. The problems related to the underestimation are probably relatively small. A more definite answer to this, however, is not possible to give as long as "real" independent date are not available.

Logistic model for prediction of mortality for individual trees have been developed. The model was developed from data set provided from crimean pine plantations in Turkey. Although mortality as a phenomenon is complicated to model, the model fit and the validation tests turned out satisfactory.

The mortality rate in crimean pine plantations is the best explained by the relative size of a tree (d/d_q^{-}), stand basal area and site index.

The presented mortality models seem to hold an appropriate level of reliability and they can be applied in forest management decision making activities. This does not mean that the model can not be enhanced, hoewever. With sample plot remeasurements, the model should be evaluated and if necessary, revised or calibrated.

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